Learning Concepts Definable in First-Order Logic with Counting

---

Steffen van Bergerem
RWTH Aachen University

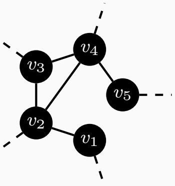
Highlights 2019

Learning Concepts Definable in Logics

Fixed

background structure

graph / relational database



Input

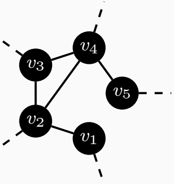
Output

Learning Concepts Definable in Logics

Fixed

background structure

graph / relational database



Input

labeled examples

$((v_1, v_2), \text{True}),\ ((v_2, v_1), \text{False}), ((v_1, v_3), \text{False}),$
$((v_2, v_3), \text{False}), ((v_2, v_4), \text{True}),\ ((v_4, v_2), \text{True})$

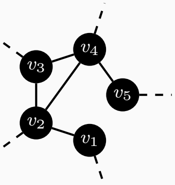
Output

Learning Concepts Definable in Logics

Fixed

background structure

graph / relational database



Input

labeled examples

$((v_1, v_2), \text{True}),\ ((v_2, v_1), \text{False}), ((v_1, v_3), \text{False}),$
$((v_2, v_3), \text{False}), ((v_2, v_4), \text{True}),\ ((v_4, v_2), \text{True})$

Output

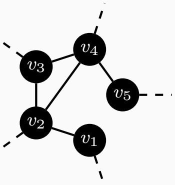
consistent parametric model

$$\varphi(\underbrace{x_1, \dots, x_k}_{\text{instance variables}} \,;\, \underbrace{y_1, \dots, y_\ell}_{\text{parameter variables}}), \quad \underbrace{w_1, \dots, w_\ell}_{\text{parameters}}$$

Learning Concepts Definable in Logics

Fixed background structure

graph / relational database



Input labeled examples

$$((v_1, v_2), \text{True}),\ ((v_2, v_1), \text{False}),\ ((v_1, v_3), \text{False}),$$
$$((v_2, v_3), \text{False}),\ ((v_2, v_4), \text{True}),\ ((v_4, v_2), \text{True})$$

Output consistent parametric model

$$\varphi(\underbrace{x_1, \dots, x_k}_{\text{instance variables}} \,;\, \underbrace{y_1, \dots, y_\ell}_{\text{parameter variables}}), \quad \underbrace{w_1, \dots, w_\ell}_{\text{parameters}}$$

$$\varphi(x_1, x_2 \,;\, y) = \ ?$$

$$w = \ ?$$

$$[\![\varphi(v_1, v_2 \,;\, w)]\!] = \text{True},$$

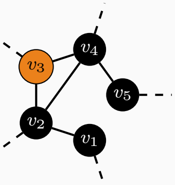
$$[\![\varphi(v_2, v_1 \,;\, w)]\!] = \text{False}, \ \dots$$

Learning Concepts Definable in Logics

Fixed

background structure

graph / relational database



Input

labeled examples

$((v_1, v_2), \text{True}),\ ((v_2, v_1), \text{False}),\ ((v_1, v_3), \text{False}),$
$((v_2, v_3), \text{False}),\ ((v_2, v_4), \text{True}),\ ((v_4, v_2), \text{True})$

Output

consistent parametric model

$$\varphi(\underbrace{x_1, \dots, x_k}_{\text{instance variables}} \ ;\ \underbrace{y_1, \dots, y_\ell}_{\text{parameter variables}}), \quad \underbrace{w_1, \dots, w_\ell}_{\text{parameters}}$$

$$\varphi(x_1, x_2 \ ;\ y) = \text{“}x_1, x_2, y \text{ is a walk”}$$

$$w = v_3$$

$$[\![\varphi(v_1, v_2 \ ;\ v_3)]\!] = \text{True},$$

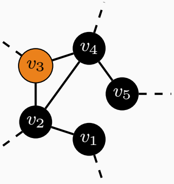
$$[\![\varphi(v_2, v_1 \ ;\ v_3)]\!] = \text{False}, \ \dots$$

Learning Concepts Definable in Logics

Fixed

background structure

graph / relational database



Input

labeled examples

$((v_1, v_2), \text{True}),\ ((v_2, v_1), \text{False}),\ ((v_1, v_3), \text{False}),$
$((v_2, v_3), \text{False}),\ ((v_2, v_4), \text{True}),\ ((v_4, v_2), \text{True})$

Output

consistent parametric model

$$\varphi(\underbrace{x_1, \ldots, x_k}_{\text{instance variables}} \,;\, \underbrace{y_1, \ldots, y_\ell}_{\text{parameter variables}}), \quad \underbrace{w_1, \ldots, w_\ell}_{\text{parameters}}$$

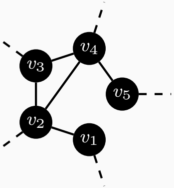
$$\varphi(x_1, x_2 \,;\, y) = Ex_1x_2 \wedge Ex_2y$$

$$w = v_3$$

$$[\![\varphi(v_1, v_2 \,;\, v_3)]\!] = \text{True},$$

$$[\![\varphi(v_2, v_1 \,;\, v_3)]\!] = \text{False}, \ \ldots$$

Learning Concepts Definable in FO

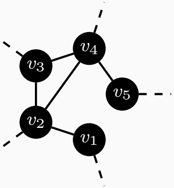


graph with $\boldsymbol{n}$ vertices
maximum degree $\boldsymbol{d}$

$((v_1, v_2), \text{True}),\ ((v_2, v_1), \text{False}), ((v_1, v_3), \text{False})$

$\boldsymbol{t}$ examples

Learning Concepts Definable in FO



graph with $\boldsymbol{n}$ vertices
maximum degree $\boldsymbol{d}$

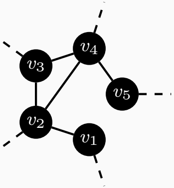
$((v_1, v_2), \text{True}),\ ((v_2, v_1), \text{False}), ((v_1, v_3), \text{False})$

$\boldsymbol{t}$ examples

Theorem (Grohe and Ritzert, 2017)

Concepts definable in FO on structures of small degree can be learned in time sublinear in $\boldsymbol{n}$ and polynomial in $\boldsymbol{t}$.

Learning Concepts Definable in FOCN(P)



graph with $\boldsymbol{n}$ vertices
maximum degree $\boldsymbol{d}$

$((v_1, v_2), \text{True}),\ ((v_2, v_1), \text{False}),\ ((v_1, v_3), \text{False})$

$\boldsymbol{t}$ examples

Theorem (Grohe and Ritzert, 2017)

Concepts definable in FO on structures of small degree can be learned in time sublinear in $\boldsymbol{n}$ and polynomial in $\boldsymbol{t}$.

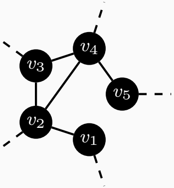
FOCN(P)

Adds counting on top of FO.

Example: $\#(\bar{x}).\varphi(\bar{x}) = \#(\bar{y}).\psi(\bar{y}) + 4$

(Kuske and Schweikardt, 2017)

Learning Concepts Definable in FOCN(P)



graph with n vertices
maximum degree d

$((v_1, v_2), \text{True}),\ ((v_2, v_1), \text{False}), ((v_1, v_3), \text{False})$

t examples

Theorem (Grohe and Ritzert, 2017)

Concepts definable in FO on structures of small degree can be learned in time sublinear in n and polynomial in t.

FOCN(P)

Adds counting on top of FO.

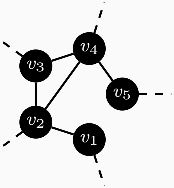
Example: $\#(\bar{x}).\varphi(\bar{x}) = \#(\bar{y}).\psi(\bar{y}) + 4$

(Kuske and Schweikardt, 2017)

Theorem

Concepts definable in FOCN(P) on structures of small degree can be learned in time sublinear in n and polynomial in t.

Learning Concepts Definable in FOCN(P) — Summary



graph with n vertices
maximum degree d

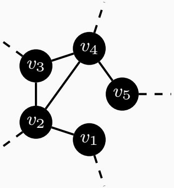
$((v_1, v_2), \text{True}),\ ((v_2, v_1), \text{False}), ((v_1, v_3), \text{False})$

t examples

Theorem

Concepts definable in FOCN(P) on structures of small degree can be learned in time sublinear in n and polynomial in t.

Learning Concepts Definable in FOCN(P) — Summary



graph with n vertices
maximum degree d

$((v_1, v_2), \text{True}),\ ((v_2, v_1), \text{False}),\ ((v_1, v_3), \text{False})$

t examples

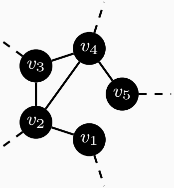
Theorem

Concepts definable in FOCN(P) on structures of small degree can be learned in time sublinear in n and polynomial in t.

Theorem

Learning concepts definable in FO on structures of unbounded degree is not possible in sublinear time.

Learning Concepts Definable in FOCN(P) — Summary



graph with n vertices
maximum degree d

$((v_1, v_2), \text{True}),\ ((v_2, v_1), \text{False}), ((v_1, v_3), \text{False})$

t examples

Theorem

Concepts definable in FOCN(P) on structures of small degree can be learned in time sublinear in n and polynomial in t.

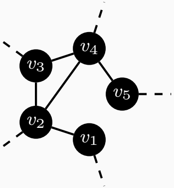
Theorem

Learning concepts definable in FO on structures of unbounded degree is not possible in sublinear time.

Theorem

Learning parameters for concepts definable in FO with quantifier rank q on structures of unbounded degree is not possible in time $n^{o(q)}$. (assuming ETH)

Learning Concepts Definable in FOCN(P) — Summary



graph with $\boldsymbol{n}$ vertices
maximum degree $\boldsymbol{d}$

$((v_1, v_2), \text{True}),\ ((v_2, v_1), \text{False}), ((v_1, v_3), \text{False})$

$\boldsymbol{t}$ examples

Not presented:
PAC-learning

Theorem

Concepts definable in FOCN(P) on structures of small degree can be learned in time sublinear in $\boldsymbol{n}$ and polynomial in $\boldsymbol{t}$.

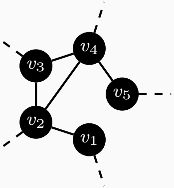
Theorem

Learning concepts definable in FO on structures of unbounded degree is not possible in sublinear time.

Theorem

Learning parameters for concepts definable in FO with quantifier rank q on structures of unbounded degree is not possible in time $\boldsymbol{n}^{\boldsymbol{o(q)}}$. (assuming ETH)

Learning Concepts Definable in FOCN(P) — Summary



graph with n vertices
maximum degree d

$((v_1, v_2), \text{True}),\ ((v_2, v_1), \text{False}), ((v_1, v_3), \text{False})$

t examples

Not presented:
PAC-learning

Theorem

Concepts definable in FOCN(P) on structures of small degree can be learned in time sublinear in n and polynomial in t.

Other aggregators
from SQL?

Theorem

Learning concepts definable in FO on structures of unbounded degree is not possible in sublinear time.

Better lower bounds?

Theorem

Learning parameters for concepts definable in FO with quantifier rank q on structures of unbounded degree is not possible in time $n^{o(q)}$. (assuming ETH)